NAG Toolbox for MATLAB

f07bs

1 Purpose

f07bs solves a complex band system of linear equations with multiple right-hand sides,

$$AX = B, \quad A^{\mathrm{T}}X = B \quad \text{or} \quad A^{\mathrm{H}}X = B,$$

where A has been factorized by f07br.

2 Syntax

```
[b, info] = f07bs(trans, kl, ku, ab, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07bs is used to solve a complex band system of linear equations $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$, the function must be preceded by a call to f07br which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** = 'T', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If **trans** = 'C', the solution is computed by solving $U^{\mathrm{H}}Y = B$ and then $L^{\mathrm{H}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **trans – string**

Indicates the form of the equations.

trans = 'N'

$AX = B$ is solved for X.

trans = 'T'

$A^{\mathrm{T}}X = B$ is solved for X.

trans = 'C'

$A^{\mathrm{H}}X = B$ is solved for X.

Constraint: **trans** = 'N', 'T' or 'C'.

2: **kl – int32 scalar**

k_l, the number of subdiagonals within the band of the matrix A.

Constraint: **kl** ≥ 0.

3: **ku – int32 scalar**

k_u, the number of superdiagonals within the band of the matrix A.

Constraint: $\mathbf{ku} \geq 0$.

4: **ab(ldab,$*$) – complex array**

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The LU factorization of A, as returned by f07br.

5: **ipiv($*$) – int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

The pivot indices, as returned by f07br.

6: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb

5.4 Output Parameters

1: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **trans**, 2: **n**, 3: **kl**, 4: **ku**, 5: **nrhs_p**, 6: **ab**, 7: **ldab**, 8: **ipiv**, 9: **b**, 10: **ldb**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(k)\epsilon|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll n$.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}\left(A^{\mathrm{H}}\right)$ (which is the same as $\mathrm{cond}\left(A^{\mathrm{T}}\right)$) can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07bv, and an estimate for $\kappa_\infty(A)$ can be obtained by calling f07bu with **norm_p** = 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $8n(2k_l + k_u)r$, assuming $n \gg k_l$ and $n \gg k_u$.

This function may be followed by a call to f07bv to refine the solution and return an error estimate.

The real analogue of this function is f07be.

9 Example

```
trans = 'N';
m = int32(4);
kl = int32(1);
ku = int32(2);
ab = [complex(0, 0), complex(0, 0), complex(0, 0), complex(0, 0);
      complex(0, 0), complex(0, 0), complex(0.97, -2.84), complex(0.59, -
0.48);
         complex(0,  0),  complex(-2.05,  -0.85),  complex(-3.99,  +4.01),
complex(3.33, -1.04);
     complex(-1.65, +2.26), complex(-1.48, -1.75), complex(-1.06, +1.94),
complex(-0.46, -1.72);
        complex(0,  +6.3),  complex(-0.77,  +2.83),  complex(4.48,  -1.09),
complex(0, 0)];
b = [complex(-1.06, +21.5), complex(12.85, +2.84);
     complex(-22.72, -53.9), complex(-70.22, +21.57);
     complex(28.24, -38.6), complex(-20.73, -1.23);
     complex(-34.56, +16.73), complex(26.01, +31.97)];
```

```
[ab, ipiv, info] = f07br(m, kl, ku, ab);
[bOut, info] = f07bs(trans, kl, ku, ab, ipiv, b)
```

```
bOut =
  -3.0000 + 2.0000i   1.0000 + 6.0000i
   1.0000 - 7.0000i  -7.0000 - 4.0000i
  -5.0000 + 4.0000i   3.0000 + 5.0000i
   6.0000 - 8.0000i  -8.0000 + 2.0000i
info =
           0
```